

Fig. 2 Variation of the square of the \bar{x} component near separation.

($Ax = 0.05$) for various values of the dimensionless time, Bt . As time increases and separation moves closer to this station, the velocity first becomes negative near the wall and then the region of reverse flow increases in size.

For this case, as the solution in the moving coordinate system approached $A\bar{x} \cong 0.3158$ the number of iterations increased rapidly and at $A\bar{x} = 0.3158$ it was not possible to obtain a converged solution. This was taken as an indication of a singularity in the vicinity of this point. As this point is approached the velocity profiles in the moving coordinate system approach a velocity profile for which the velocity and the shear vanish simultaneously at a point within the boundary layer.

Figure 2 presents the variation of the square of the velocity, as seen in the moving coordinate system, as a function of the distance from the separation point at the fixed value of $y[AU_\infty/\nu]^{1/2} = 1.881$ which corresponds to the minimum velocity in the last station for which convergence was obtained. It is clear that near separation the velocity approaches zero as $u^2 \sim (\bar{x}_{sep} - \bar{x})$ indicating a Goldstein-type square root singularity as postulated by Moore¹ and Telionis.⁶ Another indication of this singular behavior is given in Fig. 3 when the normalized y component of velocity $[v/(U_\infty \nu A)^{1/2}]$ is presented as a function of ξ for

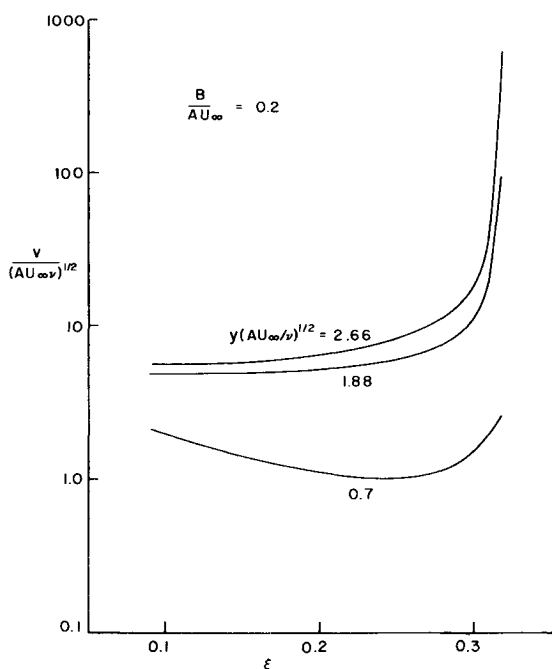


Fig. 3 Variation of the y component of velocity with ξ for several values of $y(AU_\infty/\nu)^{1/2}$.

several values of $y(AU_\infty/\nu)^{1/2}$, the normalized y coordinate. In the vicinity of $y(AU_\infty/\nu)^{1/2} = 1.881$ and for values of $y(AU_\infty/\nu)^{1/2}$ greater than this the y component of velocity increases rapidly as separation is approached, as one would expect for a Goldstein-type singularity. Near the wall, however, the y component of velocity does not appear to behave in a singular manner, indicating again that the singular point lies away from the wall.

The behavior of the flow indicated above verifies the Moore-Rott-Sears model for unsteady separation in that the separation point is characterized by: 1) the simultaneous vanishing of the shear and velocity at a point in the boundary layer in the flow "... seen by an observer moving with the separation point," and 2) a singular behavior of the boundary-layer equations at this point.

References

- 1 Moore, F. K., "On the Separation of the Unsteady Laminar Boundary Layer," in *Boundary Layer Research*, edited by H. G. Görtler, Springer-Verlag, Berlin, 1958, pp. 296-310.
- 2 Rott, N., "Unsteady Flow in the Vicinity of a Stagnation Point," *Quarterly of Applied Mathematics*, Vol. 13, 1956, pp. 444-451.
- 3 Sears, W. R., "Some Recent Developments in Airfoil Theory," *Journal of the Aeronautical Sciences*, Vol. 23, No. 5, 1956, pp. 490-499.
- 4 Rott, N., "Theory of Time-Dependent Laminar Flows," in *Theory of Laminar Flows*, edited by F. K. Moore, Princeton Univ. Press, Princeton, N.J., 1964, p. 432.
- 5 Sears, W. R. and Telionis, D. P., "Unsteady Boundary Layer Separation," in *Fluid Dynamics of Unsteady, Three-Dimensional and Separated Flows*, Proceedings of a SQUID Workshop, edited by F. J. Marshall, Atlanta, Ga., June 1971.
- 6 Telionis, D. P., "Boundary Layer Separation," Ph.D. thesis, Sept. 1970, Cornell University, Ithaca, N.Y.
- 7 Blottner, F. G., "Finite-Difference Methods of Solution of the Boundary Layer Equations," *AIAA Journal*, Vol. 8, No. 2, Feb. 1970, pp. 193-205.
- 8 Telionis, D. P. and Werle, M. J., "Boundary Layer Separation from Downstream Moving Boundaries," *Journal of Applied Mechanics*, Vol. 95, June 1973, pp. 369-374.

Burning Constant-Stoichiometric Ratio Relation—Some Clarifications

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ESSENHIGH mentions in his paper¹ with reference to burning of polymer spheres that a correlation between the burning constant (or evaporation constant) and oxidizer to fuel ratio can be found. This point has been reasserted by Essenhigh and Dreier.² This Note is primarily intended to point out that such a correlation should not be expected.

The burning constant K is defined by

$$t = K(d_o^2 - d^2) \quad (1)$$

where d_o = initial diameter, d = diameter at any time t . This correlation is inferred from an analysis of droplet combustion by neglecting the convection terms in the conservation relations. Such an analysis leads to

$$\frac{d_f}{d} = 1 + \frac{\rho_f \mathcal{D}_f}{\rho_o \mathcal{D}_o} i \quad (2)$$

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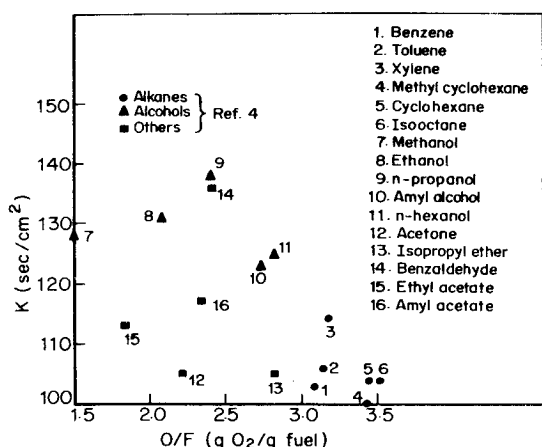


Fig. 1 Burning constant vs oxidizer/fuel ratio for liquid droplets.

where d_f is the flame diameter, \mathcal{D}_f , \mathcal{D}_o are the diffusion coefficients of fuel and oxidizer, ρ_f , ρ_o are the vapor densities of fuel and oxidizer, p_o is the partial pressure of oxidizer in ambient air, and i is the stoichiometric ratio ($= O/F$).

Now the usual theory³ of liquid droplet combustion including convective effects leads to

$$K \propto (1 - d/d_f) \quad (3)$$

Combining Eqs. (2) and (3), Essenhigh and Dreier get

$$K \propto \frac{O/F}{\alpha + O/F} \quad (4)$$

where $\alpha = \mathcal{D}_o p_o \rho_o / \mathcal{D}_f \rho_f$. Relation (4) has been used to justify the plot of K vs O/F . If we use the usual theory of liquid droplets for obtaining d_f/d , we get

$$\frac{d_f}{d} = \frac{\ln(1+B)}{\ln[1 + (p_o/i)(M_o/pM)]} \quad (5)$$

where B is the well-known transfer number, M_o and M are the molecular weights of oxidant and mixture, and p the total pressure. It can now be noticed that the more correct result (5) (only "more correct" since free convection and other effects have not been included in both cases) does not lead in any limit to Eq. (2). Also the exact result for K given by

$$K = \rho_l / 8 \bar{\mathcal{D}} \rho \ln(1+B) \quad (6)$$

where ρ_l = density of polymer, $\bar{\mathcal{D}}$ = molecular diffusivity, and ρ = density in the gas phase, does not produce Eq. (4) in any appropriate limit.

Suspecting that the correlation K vs O/F would be inappropriate, it was thought fit to check the correlation against known results on liquid droplets. It was particularly inviting to perform this since Essenhigh and Dreier also draw upon the conclusions from liquid droplet burning. The results of K vs O/F

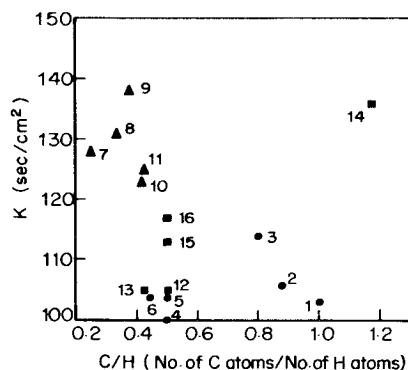


Fig. 2 Burning constant vs C/H ratio for liquid droplets.

for a member of alkanes, alcohols, and others⁴ have been plotted in Fig. 1. It is apparent that no trace of correlation can be seen. The only general conclusion appears to be that oxygen containing substances have larger K (smaller burning rate) than others.

Essenhigh and Dreier also attempt to invoke a correlation between K and C/H ratio on the basis that such a correlation exists for liquid droplets. Figure 2 shows the plot of K vs C/H for the same liquids cited in Fig. 1. Again the absence of any regular behavior is more than evident.

The liquid droplet theory by Godsavage³ worked out as long ago as 1953, is quite simple and contains the principal features of droplet combustion. The result for K as in Eq. (6) has been verified to be accurate by several investigators.⁴ While the authors² use Godsavage's theory to evolve Eq. (3), they use an approximate theory, neglecting the convection terms, to obtain Eq. (2) and consequently arrive at incorrect relations and conclusions.

References

- Essenhigh, R. H., "On the Inter-Influence of Classical Heterogeneous Combustion Research and Related Aerospace Problems," *Eleventh International Symposium on Combustion*, The Combustion Institute, Pittsburgh, Pa., 1967, pp. 291-308.
- Essenhigh, R. H. and Dreier, W. L., "Combustion Behaviour of Thermoplastic Polymer Spheres Burning in Quiescent Atmosphere of Air," *Fuel*, Vol. 48, No. 4, 1969, pp. 330-342.
- Godsavage, G. A. E., "Studies of the Combustion of Drops in a Fuel Spray—The Burning of Single Drops of Fuel," *Fourth Symposium (International) on Combustion*, Williams and Wilkins, Baltimore, Md., 1953, pp. 818-830.
- Hedley, A. B., Nuruzzaman, A. S. M., and Martin, G. F., "Progress Review No. 62: Combustion of Single Droplets and Simplified Spray Systems," *Journal of the Institute of Fuel*, Vol. 44, No. 360, 1971, pp. 38-54.

Transformation for the Numerical Integration of Systems of the Form

$$\ddot{X}(t) = F[X(t)]$$

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MANY problems can be characterized by a second-order differential equation of the form

$$\ddot{X}(t) = F[X(t)] \quad (1)$$

where $X(t)$, $F[X(t)]$ are each n -vectors, and $F[X(t)]$ has a continuous partial derivative with respect to $X(t)$, for $X(t)$ in some subset of Euclidean n -space. Such differential equations may arise from Newton's second law of motion, or they may describe a two-point boundary-value problem. For numerical integration from t_0 to t_1 with known initial conditions $X(t_0)$, $\dot{X}(t_0)$, Eq. (1) is usually converted to the equivalent first-order system

$$\dot{X}(t) = Y(t) \quad (2)$$

$$\dot{Y}(t) = F[X(t)] \quad (3)$$

It is known¹ that the errors in the numerical integration of Eqs. (2) and (3) depend on the eigenvalues of the matrix

$$H(t) = \begin{bmatrix} \Theta & \vdots & I \\ J[X(t)] & \vdots & \Theta \end{bmatrix} \quad (4)$$

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